FIN 538

Problem Set 1 – Solutions

① a. $y_2 = 0.1 + 0.9\,y_1 = 2.8$

$y_3 = 0.1 + 0.9\,(0.1 + 0.9\,y_1)$

$= 0.1 + 0.9 \cdot 0.1 + 0.9^2\,y_1$

$= 0.1\,(1 + 0.9^1) + 0.9^2\,y_1 = 2.62$

$\vdots$

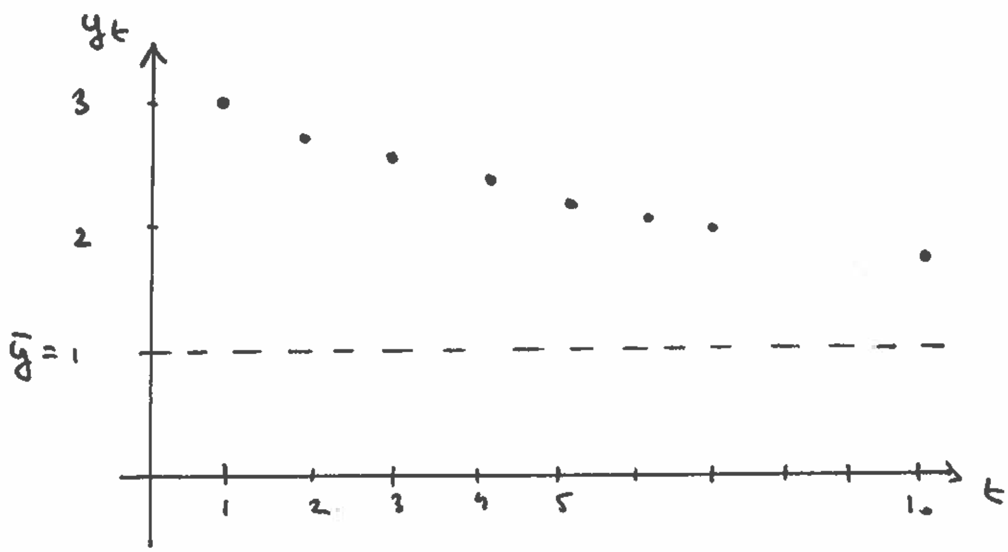
$y_{10} = 0.1\,(1 + 0.9 + 0.9^2 + \dots + 0.9^8) + 0.9^9\,y_1$

$= 0.1\left(\dfrac{1-0.9^9}{1-0.9}\right) + 0.9^9\,y_1$

$= 0.1\left(\dfrac{1-0.9^9}{1-0.9}\right) + 0.9^9 \times 3$

$= 1.775$

b. $\bar{y} = 0.1 + 0.9\,\bar{y} \Rightarrow \bar{y} = 1$

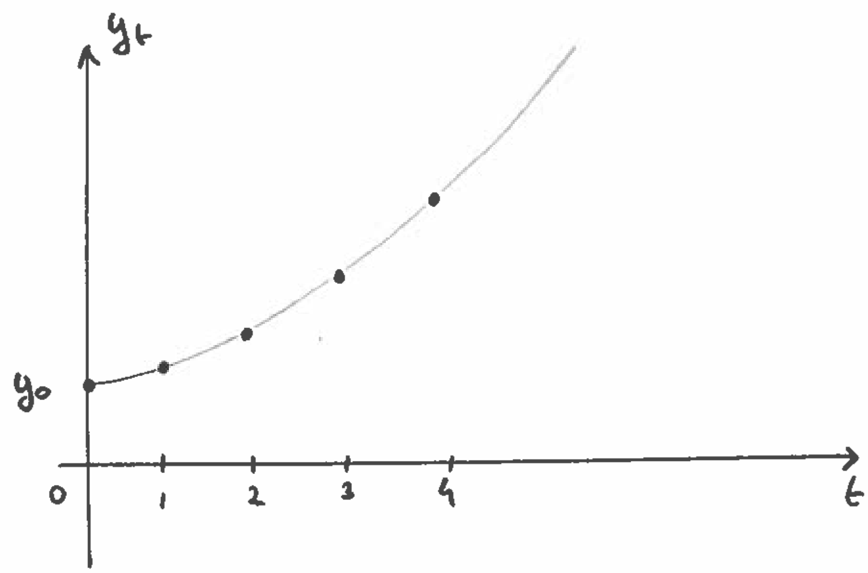


②

a. $x_{t+1} = a + x_t$

$$\Rightarrow \quad x_1 = a + x_0$$
$$x_2 = a + (a + x_0) = 2a + x_0$$
$$\vdots$$
$$x_t = at + x_0$$
$$y_t = \exp(at + x_0) = y_0 e^{at}$$

b.



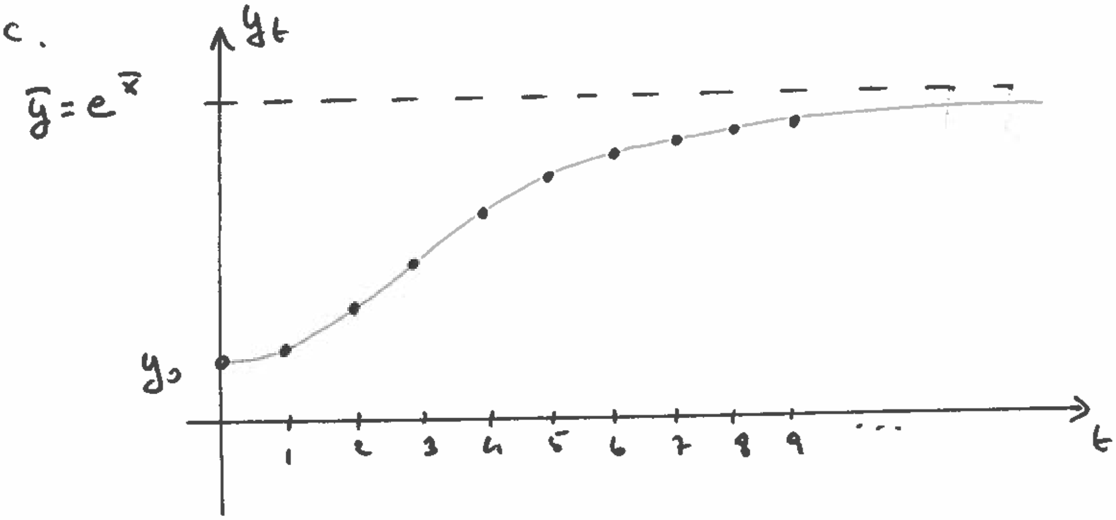
(3) a. If $0 < x_0 < \kappa$, the growth rate of x_t given by $r\left(1 - \frac{x_t}{\kappa}\right)$ is positive. This implies that x_t increases with time. However, as this happens the growth rate decreases. Eventually, as $x_t \to \kappa$ we have that $r\left(1 - \frac{x_t}{\kappa}\right) \longrightarrow 0$.

b. In steady state, we must have that

$$\bar{x} = \bar{x}\left(1 + r\left(1 - \frac{\bar{x}}{\kappa}\right)\right)$$

$$\Rightarrow 1 - \frac{\bar{x}}{\kappa} = 0 \Rightarrow \bar{x} = \kappa.$$

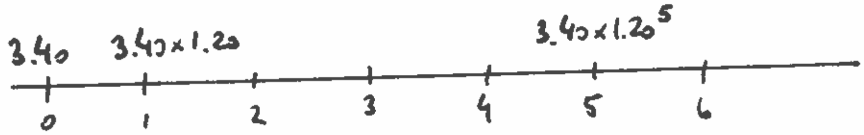
c.



The graph in this case does not explode but converges to $\bar{y} = e^{\bar{x}}$.

(4)

a.



$$V = \frac{3.40 \times 1.20}{0.10 - 0.20}\left[1 - \left(\frac{1.20}{1.10}\right)^4\right] + \frac{3.40 \times 1.20^5}{0.10 - 0.03} \times \frac{1}{1.10^4}$$

$$= 99.49$$

b. No, because the dividends paid by the stock do not change nor the discount rate.

(5) a. $F_0 = 0$ $F_1 = 1$ $F_2 = 1$ $F_3 = 2$ $F_4 = 3$ $F_5 = 5$

$F_6 = 8$ $F_7 = 13$ $F_8 = 21$ $F_9 = 34$ $F_{10} = 55$

b. Clearly not since we keep adding larger numbers together.

c. Let $G_n = F_{n-1}$. Then

$$\begin{aligned} F_n &= F_{n-1} + G_{n-1} \\ G_n &= G_{n-1} \end{aligned} \Rightarrow \begin{pmatrix} F_n \\ G_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} F_{n-1} \\ G_{n-1} \end{pmatrix}$$

where $\begin{pmatrix} F_1 \\ G_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

⑥

a. The impulse response is given by

$$x_{-1} = 0$$

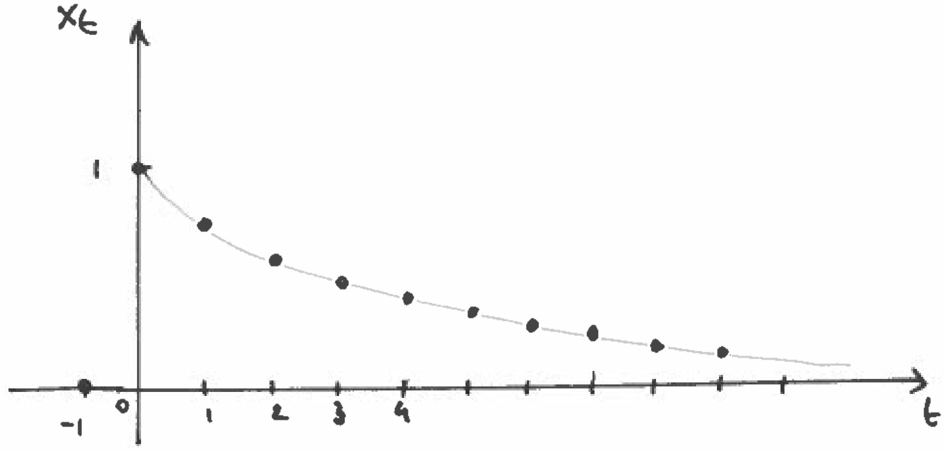
$$x_0 = 0.95\, x_{-1} + 1 = 1$$

$$x_1 = 0.95 \times 1 + 0 = 0.95$$

$$x_2 = 0.95 \times 0.95 + 0 = 0.95^2$$

$$\vdots$$

$$x_t = 0.95^t$$



b. $$x_t = 0.95^t = 0.5$$

$$t \ln 0.95 = \ln 0.5$$

$$t = 13.51$$

$$0.95^{13} = 0.513$$

$$0.95^{14} = 0.488 \Rightarrow t = 14 \text{ is the closest.}$$

7 a. $$X_t = X_{t-1}(1+r) - r X_{t-1} = X_{t-1}$$

b. Let $c = r X_0$

Then $X_0 = \frac{c}{r} = X_t$ for $t \geq 0$,

is the present value of the perpetuity.